PERIOD-I

M10CH2

TOPIC

2

Rational Numbers

2.1 ADDITION *AND* SUBTRACTION OF RATIONAL NUMBERS

Identifying Rational Numbers

The word 'rational' comes from the word 'ratio'. Thus, rational number arises as a ratio of two integers (excluding '0' in the denominator).

A rational number is defined as *a number that can be expressed in the form* $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

ACTIVITY 1

Consider $\frac{3}{7}$ is a rational number.

Here, $a = 3$ and $b = 7$.

Is $\frac{-4}{5}$ also a rational number?

For example:

(*a*) $6 = \frac{6}{1}$ (*b*) $0 = \frac{0}{1}$ (*c*) $-8 = -\frac{8}{1}$

(*d*) $\frac{2}{3}$ (*e*) $\frac{-9}{5}$ (*f*) $\frac{7}{13}$

All of these numbers are rational numbers.

Thus, *rational numbers include integers and fractions.*

Example 1. *Identify which of the following are rational numbers and give reasons:*

(*a*) $\frac{2}{7}$ (*b*) $\frac{-4}{3}$ (*c*) $\frac{0}{5}$ (*d*) $\frac{11}{0}$.

Solution. (*a*) $\frac{2}{7}$ is a rational number since 2 and 7 are integers, and $7 \neq 0$.

(*b*) $\frac{-4}{3}$ is a rational number since –4 and 3 are integers and $3 \neq 0$.

(*c*) $\frac{0}{5}$ is a rational number since 0 and 5 are integers and $5 \neq 0$.

(*d*) $\frac{11}{0}$ is not a rational number. Though 11 and 0 are integers, the denominator is zero and division by zero has no meaning.

Addition of Rational Numbers

While adding rational numbers with same denominators, we add the numerators keeping the denominators same.

For example: $\frac{-11}{5} + \frac{7}{5} = \frac{-11+7}{5} = \frac{-4}{5}$

How do we add rational numbers with different denominators?

As in the case of fractions, we first find the LCM of the two denominators. Then, we find the equivalent rational numbers of the given rational numbers with this LCM as the denominator. Then, add the two rational numbers.

For example: Let us add $\frac{-7}{5}$ and $\frac{-2}{3}$.

LCM of 5 and 3 is 15.

So, $\frac{-7}{5} = \frac{-7}{5} \times \frac{3}{3} = \frac{-21}{15}$

and $\frac{-2}{3} = \frac{-2}{3} \times \frac{5}{5} = \frac{-10}{15}$.

Thus, $\frac{-7}{5} + \frac{(-2)}{3} = \frac{-21}{15} + \frac{(-10)}{15} = \frac{-31}{15}$

Subtraction of Rational Numbers

Additive Inverse

What will be $\frac{-4}{7} + \frac{4}{7} = ?$

$$\frac{-4}{7} + \frac{4}{7} = \frac{-4+4}{7} = 0.$$

Also, $$\frac{4}{7} + \left(\frac{-4}{7}\right) = 0.$$

Similarly, $$\frac{-2}{3} + \frac{2}{3} = 0 = \frac{2}{3} + \left(\frac{-2}{3}\right).$$

In the case of integers, we call –2 as the additive inverse of 2 and 2 as the additive inverse of –2.

For rational numbers also, we call $\frac{-4}{7}$ as the *additive inverse* of $\frac{4}{7}$ and $\frac{4}{7}$ as the additive inverse of $\frac{-4}{7}$.

ACTIVITY 2

Islah found the difference of two rational numbers $\frac{5}{7}$ and $\frac{3}{8}$ in this way:

$$\frac{5}{7} - \frac{3}{8} = \frac{40}{56} - \frac{21}{56} = \frac{40-21}{56} = \frac{19}{56}$$

Aaliyah knew that for two integers a and b she could write

$$a - b = a + (-b)$$

She tried this for rational numbers also and found out that:

$$\frac{5}{7} - \frac{3}{8} = \frac{5}{7} + \frac{(-3)}{8} = \frac{40}{56} + \frac{(-21)}{56} = \frac{19}{56}.$$

Are Islah and Aaliyah both obtained the same difference?

Try to find $\frac{7}{8}-\frac{5}{9}$ and $\frac{3}{11}-\frac{8}{7}$ in both ways.

Did you get the same answer?

So, we say *while subtracting two rational numbers, we add the additive inverse of the rational number that is being subtracted, to the other rational number.*

Thus, $\frac{5}{3}-\frac{14}{5}=\frac{5}{3}$ + additive inverse of $\frac{14}{5}$

$$= \frac{5}{3}+\frac{(-14)}{5} = \frac{25}{15}+\frac{(-42)}{15} = \frac{-17}{15}.$$

What will be $\frac{2}{7}-\left(\frac{-5}{6}\right)$?

$$\frac{2}{7}-\left(\frac{-5}{6}\right) = \frac{2}{7} + \text{additive inverse of} \left(\frac{-5}{6}\right)$$

$$= \frac{2}{7}+\frac{5}{6} = \frac{12}{42}+\frac{35}{42} = \frac{47}{42}$$

Example 2. *Find:* (a) $\frac{-7}{3}+\frac{23}{5}$ (b) $-\frac{19}{9}-6$.

Solution. (a) $\frac{-7}{3}+\frac{23}{5}$

LCM of 3 and 5 is 15

So, $\frac{-7}{3}=\frac{-35}{15}$ and $\frac{23}{5}=\frac{69}{15}$

Thus, $\frac{-7}{3}+\frac{23}{5}=\frac{-35}{15}+\frac{69}{15}=\frac{34}{15}$

(b) $\frac{-19}{9}-6=\frac{-19}{9}-\frac{54}{9}=\frac{-19-54}{9}=\frac{-73}{9}$.

2.2 MULTIPLICATION OF RATIONAL NUMBERS

While multiplying a rational number by an integer (positive or negative), we multiply the numerator by that integer, keeping the denominator unchanged.

Let us now multiply a rational number by a negative integer,

For example: $\frac{-2}{9} \times (-5) = \frac{-2 \times (-5)}{9} = \frac{10}{9}$

Remember, –5 can be written as $\frac{-5}{1}$.

So, $\frac{-2}{9} \times \frac{-5}{1} = \frac{-2 \times (-5)}{9 \times 1} = \frac{10}{9}$

Similarly, $\frac{3}{11} \times (-2) = \frac{3 \times (-2)}{11 \times 1} = \frac{-6}{11}$.

Based on these observations, we find that,

$$\frac{-3}{8} \times \frac{5}{7} = \frac{-3 \times 5}{8 \times 7} = \frac{-15}{56}.$$

So, as we did in the case of fractions, we multiply two rational numbers using the following steps:

Step 1. Multiply the numerators of the two rational numbers.

Step 2. Multiply the denominators of the two rational numbers.

Step 3. Write the product as $\frac{\text{Result of step 1}}{\text{Result of step 2}}$

Thus $\frac{-3}{5} \times \frac{2}{7} = \frac{-3 \times 2}{5 \times 7} = \frac{-6}{35}$.

Also, $\frac{-5}{8} \times \frac{-9}{7} = \frac{-5 \times (-9)}{8 \times 7} = \frac{45}{56}$.

Note 1. The product of two negative numbers is a positive number, *e.g.,* –2 × –2 = 4.

2. The product of a negative and a positive number is a negative number, *e.g.,* –2 × 2 = –4.

3. The product of two positive numbers is a positive number, *e.g.,* 2 × 2 = 4.

EXERCISE 2.1

1. Is the number $\frac{2}{-3}$ rational? Why?
2. Do rational numbers include all fractions?
3. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?
4. Find:

   (a) $\frac{-13}{7}+\frac{6}{7}$ (b) $\frac{19}{5}+\left(\frac{-7}{5}\right)$ (c) $\frac{-3}{7}+\frac{2}{3}$ (d) $\frac{-5}{6}+\frac{-3}{11}$
5. What will be the additive inverse of the rational numbers given below?

   (a) $\frac{-3}{9}$ (b) $\frac{-9}{11}$
6. Find: (a) $\frac{7}{9}-\frac{2}{5}$ (b) $\frac{11}{5}-\frac{(-1)}{3}$.
7. Find: (a) $\frac{-3}{5}\times 7$ (b) $\frac{-6}{5}\times(-2)$.

2.3 PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

(a) **Closure Property:** If a and b are two rational numbers, then $a \times b$ is also a rational number.

For example: For two rational numbers $\frac{3}{4}$ and $\frac{-1}{5}$,

$$\frac{3}{4}\times\frac{(-1)}{5}=\frac{(-3)}{20}, \text{ which is a rational number.}$$

Hence, rational numbers are *closed* under multiplication.

(b) **Associative Property:** If a, b and c are three rational numbers, then

$$a \times (b \times c) = (a \times b) \times c$$

For example: For three rational numbers $\frac{-1}{2}$, $\frac{5}{4}$ and $\frac{-6}{7}$,

$$\frac{-1}{2}\times\left[\frac{5}{4}\times\frac{(-6)}{7}\right]=\frac{-1}{2}\times\frac{(-15)}{14}=\frac{15}{28}$$

and $\left[\frac{-1}{2}\times\frac{5}{4}\right]\times\frac{(-6)}{7} = \frac{-5}{8}\times\frac{(-6)}{7} = \frac{15}{28}$.

Hence, multiplication is *associative* for rational numbers.

(*c*) **Commutative Property:** For two rational numbers a and b,

$$a \times b = b \times a$$

For example: For two rational numbers, $\frac{3}{5}$ and $\frac{(-10)}{11}$

$$\frac{3}{5}\times\frac{(-10)}{11} = \frac{-6}{11} \quad \text{and} \quad \frac{(-10)}{11}\times\frac{3}{5} = \frac{-6}{11}$$

Hence, multiplication is commutative for rational numbers.

(*d*) **Multiplicative Identity:** When a rational number a is multiplied by 1, the product is the rational number itself *i.e.* $a \times 1 = 1 \times a = a$

For example: (*i*) $\frac{-3}{7}\times 1 = \frac{-3}{7}$ (*ii*) $1\times\frac{4}{5} = \frac{4}{5}$

We say that 1 is the multiplicative *identity* for rational numbers.

(*e*) **Multiplicative Inverse:** The multiplicative inverse for a rational number is its reciprocal.

Let a be a rational number. Then,

$$a \times \frac{1}{a} = 1$$

The product of a rational number and its multiplicative inverse is 1.

Note: 0 has no multiplicative inverse because $\frac{1}{0}$ is not defind.

For example: Consider a rational number $\frac{4}{9}$. Its reciprocal is $\frac{9}{4}$.

$$\therefore \quad \frac{4}{9}\times\frac{9}{4} = 1$$

Remark: Reciprocal of 1 is 1. • Reciprocal of –1 is –1. • Zero has no reciprocal.

(*f*) **Property of 0:** Every rational number when multiplied by 0, gives the product 0.

Hence, for a rational number a

$$a \times 0 = 0$$

Example 3. *Verify:* $\left(\frac{-3}{7} \times \frac{4}{5}\right) \times \frac{-5}{9} = \frac{4}{5} \times \left(\frac{-3}{7} \times \frac{-5}{9}\right)$.

Solution. $\text{LHS} = \left(\frac{-3}{7} \times \frac{4}{5}\right) \times \frac{-5}{9} = \frac{-12}{35} \times \frac{-5}{9} = \frac{4}{21}$ ...(1)

$\text{RHS} = \frac{4}{5} \times \left(\frac{-3}{7} \times \frac{-5}{9}\right) = \frac{4}{5} \times \frac{15}{63} = \frac{4}{21}$ ...(2)

From (1) and (2), LHS = RHS

This proves the associative property of multiplication.

Example 4. *Find the value of x, if* $2 \times (x \times 5) = (2 \times 3) \times 5$.

Solution. By associative property of multiplication, we have

$$a \times (b \times c) = (a \times b) \times c$$

Here, $2 \times (x \times 5) = (2 \times 3) \times 5$

By associative property, we have

$$x = 3$$

Example 5. *Find the multiplicative inverse of the following:*

(*i*) $-1 \times \frac{-2}{5}$ (*ii*) -1

Solution. (*i*) We have $-1 \times \frac{-2}{5} = \frac{2}{5}$

$\therefore$ Multiplicative inverse of $\frac{2}{5}$ is $\frac{5}{2}$

$\because$ $\frac{2}{5} \times \frac{5}{2} = 1$

(*ii*) Multiplicative inverse of –1 is –1.

Example 6. *The product of two rational numbers is* $9\frac{3}{5}$. *If one of them is* $9\frac{3}{7}$, *find the other.*

Solution. Let the 2nd rational number be x.

The 1st rational number = $9\frac{3}{7} = \frac{66}{7}$.

By the given condition,

$$x \times \frac{66}{7} = \frac{48}{5} \qquad \left[\because\ 9\frac{3}{5} = \frac{48}{5}\right]$$

$$\Rightarrow \qquad x = \frac{48}{5} \times \frac{7}{66} = \frac{56}{55} = 1\frac{1}{55}.$$

Hence, the second rational number is $1\frac{1}{55}$.

EXERCISE 2.2

1. Multiplicative inverse of 0 is: ....

2. Multiplicative inverse of $\frac{-3}{5}$ is: ....

3. Write:

(*a*) The rational number that does not have a reciprocal.

(*b*) The rational numbers that are equal to their reciprocals.

4. Find the multiplicative inverse of the following:

(*a*) -13 (*b*) $\frac{-13}{19}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{-5}{8} \times \frac{-3}{7}$

5. Name the property under multiplication used in each of the following:

(*a*) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = -\frac{4}{5}$ (*b*) $-\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times \frac{-13}{17}$

6. Verify: $\frac{-5}{9} \times \left(\frac{-2}{5} \times \frac{-3}{7}\right) = \left(\frac{-5}{9} \times \frac{-2}{5}\right) \times \frac{-3}{7}$.

7. Find x in each of the following:

(*a*) $x \times (6 \times 5) = (2 \times 6) \times 5$ (*b*) $5 \times (3 \times 4) = (5 \times 3) \times x$

8. Tell what property allows you to compute $\frac{1}{3} \times \left(6 \times \frac{4}{3}\right)$ as $\left(\frac{1}{3} \times 6\right) \times \frac{4}{3}$.

9. Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

10. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

11. By what rational number should $\frac{-8}{39}$ be multiplied to get $\frac{1}{26}$?

2.4 DIVISION OF RATIONAL NUMBERS

We have studied reciprocals of a fraction earlier.

What is the reciprocal of $\frac{2}{7}$*?*

It will be $\frac{7}{2}$. We can extend this idea of reciprocals to rational numbers too.

The reciprocal of $\frac{-2}{7}$ will be $\frac{7}{-2}$ *i.e.,* $\frac{-7}{2}$; that of $\frac{-3}{5}$ would be $\frac{-5}{3}$.

Product of Reciprocals

The product of a rational number with its reciprocal is always 1, *e.g.,* $\frac{a}{b}\times\frac{b}{a}=\frac{ab}{ab}=1$.

For example: $\frac{-4}{9}\times\left(\text{reciprocal of }\frac{-4}{9}\right)=\frac{-4}{9}\times\frac{-9}{4}=1$

Similarly, $\frac{-6}{13}\times\frac{-13}{6}=1$

Try some more examples and confirm this observation.

Let us divide a rational number $\frac{4}{9}$ by another rational number $\frac{-5}{7}$,

That is $\frac{4}{9}\div\frac{-5}{7}=\frac{4}{9}\times\frac{7}{-5}=\frac{-28}{45}$.

We used the idea of reciprocal as done in fractions.

We first divided $\frac{4}{9}$ by $\frac{5}{7}$ and got $\frac{28}{45}$.

That is $\frac{4}{9} \div \frac{-5}{7} = \frac{-28}{45}$.

How did we get that?

We divided them as fractions, ignoring the negative sign and then put the negative sign in the value so obtained.

Both approaches led to the same value $\frac{-28}{45}$. Try dividing $\frac{2}{3}$ by $\frac{-5}{7}$ both ways and see if you will get the same answer.

This shows, *to divide one rational number by the other rational number we multiply the rational number by the reciprocal of the other.*

Thus, $\frac{6}{-5} \div \frac{-2}{3} = \frac{6}{-5} \times \text{reciprocal of} \left(\frac{-2}{3}\right) = \frac{6}{-5} \times \frac{3}{-2} = \frac{18}{10}$

Example 7. *Find:*

(*a*) $\frac{-6}{5} \times \frac{9}{11}$ (*b*) $\frac{-7}{12} \div \left(\frac{-2}{13}\right)$.

Solution. (*a*) $\frac{-6}{5} \times \frac{9}{11} = \frac{-6 \times 9}{5 \times 11} = \frac{-54}{55}$

(*b*) $\frac{-7}{12} \div \left(\frac{-2}{13}\right) = \frac{-7}{12} \times \text{reciprocal of} \left(\frac{-2}{13}\right) = \frac{-7}{12} \times \frac{13}{-2} = \frac{91}{24}$.

EXERCISE 2.3

1. What will be the reciprocal of

(*a*) $\frac{-6}{11}$? (*b*) $\frac{-8}{5}$?

2. Find:

(*a*) $\frac{2}{3} \div \frac{-7}{8}$ (*b*) $\frac{-6}{7} \div \frac{5}{7}$.

2.5 DECIMAL REPRESENTATION

We know that a fraction whose denominator is 10 or 100 or 1000... etc., is called a *decimal fraction.*

Let us express some common fractions as decimal fractions:

Common Fraction	Decimal Fraction
$\frac{2}{10}$	0.2
$\frac{3}{100}$	0.03
$\frac{145}{1000}$	0.145

Identification of Terminating, Non-terminating and Repeating Decimals

A rational number can be expressed as a decimal number in two ways. By long division method or by converting the given rational number into its equivalent rational number whose denominator is 10 or 100 or 1000 ... etc.

For example:

$$\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100} = 0.80$$

$$\frac{5}{8} = \frac{5 \times 125}{8 \times 125} = \frac{625}{1000} = 0.625$$

The usual method of expressing a rational number in decimals is to carry out the long division process.

Example 8. *Express the following rational numbers in decimal forms:*

(a) $\frac{3}{4}$ *(b)* $\frac{-23}{10}$.

Solution. (*a*) Divide 3 by 4.

```
4 ) 3.00 ( 0.75
   -28
     20
    -20
      0
```

$\therefore \frac{3}{4} = 0.75$

(*b*) First divide 23 by 10.

```
10 ) 23.0 ( 2.3
    -20
      30
     -30
       0
```

Thus $\frac{23}{10} = 2.3$

$\therefore \frac{-23}{10} = -2.3$

In the above examples, we get 0 as remainder and there are finite number of digits after the decimal point. Such decimals are called *terminating decimals.*

Example 9. *Express the following rational numbers in decimal form:*

(*a*) $\frac{10}{3}$ (*b*) $\frac{1}{7}$.

Solution. (*a*) Divide 10 by 3

$$
\begin{array}{l}
3\,)\,10.000\,(\,3.333 \\
\quad -9 \\
\quad \overline{\;\;10} \\
\quad\;\; -9 \\
\quad\;\; \overline{\;\;10} \\
\quad\quad -9 \\
\quad\quad \overline{\;\;10} \\
\quad\quad\;\; -9 \\
\quad\quad\;\; \overline{\;\;\;1}
\end{array}
$$

$\therefore \quad \frac{10}{3} = 3.333...$

(*b*) Divide 1 by 7.

$$
\begin{array}{l}
7\,)\,1.000000...\,(\,0.142857 \\
\quad -7 \\
\quad \overline{\;\;30} \\
\quad -28 \\
\quad \overline{\;\;\;20} \\
\quad\;\; -14 \\
\quad\;\; \overline{\;\;\;60} \\
\quad\quad -56 \\
\quad\quad \overline{\;\;\;40} \\
\quad\quad\;\; -35 \\
\quad\quad\;\; \overline{\;\;\;50} \\
\quad\quad\quad -49 \\
\quad\quad\quad \overline{\;\;\;\;1}
\end{array}
$$

$\therefore \quad \frac{1}{7} = 0.142857...$

From the above examples, we observe that:

(*a*) The division never comes to an end.

(*b*) The block of digits repeats itself again and again.

We call such decimals a *non-terminating* and *repeating* or *non-terminating* and *recurring decimals.*

While writing in decimals, we place a bar or dot over the repeated part.

For example:

$$\frac{10}{3} = 3.333... = 3.\overline{3} = 3.\dot{3}$$

$$\frac{1}{7} = 0.142857... = 0.\overline{142857} = 0.\dot{1}4285\dot{7}$$

Hence, we can say that *every rational number can be expressed as either a terminating decimal or a non-terminating* and *repeating (recurring) decimal.*

Recognising Decimal Fractions that are Non-terminating and Non-repeating

Let us consider an example of a *non-terminating* and *non-repeating* decimal.

0.10110111011110... is a decimal which does not terminate and has no repeating part. Such numbers are called *non-terminating* and *non-repeating decimals*. They cannot be converted into exact rational numbers. So they are called *non-rational* or *irrational numbers.*

$\sqrt{2}$ and π are also non-rational numbers. Here are their decimal expansions up to a certain stage:

$$\sqrt{2} = 1.4142135623730950488016 88...$$

$$\pi = 3.14159265358979323846264338...$$

(Note that, we take $\frac{22}{7}$ as an approximate value for π, but $\pi \neq \frac{22}{7}$).

The *square root of a prime number* is an irrational number.

Since, 2, 3, 5, 7, 11, 13, 17, 19, ... are all prime numbers, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$, $\sqrt{13}$, $\sqrt{17}$, $\sqrt{19}$ are all *irrational numbers.*

Example 10. *Explain why 0.333 is a rational number and π is not.*

Solution. We have, $0.333 = \frac{333}{1000}$

So, it is a rational number.

$$\pi = 3.14159265...$$

It is a decimal which does not terminate and has no repeating part. So, it is not a rational number.

2.6 REAL NUMBERS (R)

Now, equipped with the knowledge of both, the rational as well as the irrational, if we again look at the number line to explore, if any number is left on it. The answer is emphatic no ! It turns out that:

(*i*) The entire collection of all rational numbers and irrational numbers has been picked up, and

(*ii*) No point, on the number line, is now left unrepresented by a number.

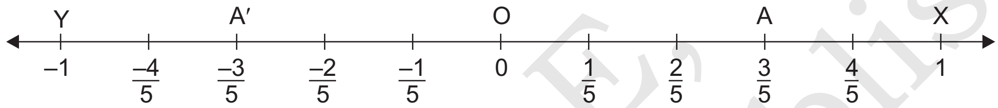
Every real number is either a rational number or an irrational number.

2.7 THE REAL NUMBER LINE

We know that on a number line of integers, the points on the right of zero represent positive integers and the points on the left of zero represent negative integers. Similarly we can represent rational numbers.

Example 11. *Represent* $\frac{3}{5}$ *and* $\frac{-3}{5}$ *on the number line.*

Solution. Draw a number line, represent zero by O. At equal distances from O mark 1 by X and –1 by Y. Divide OX and OY into 5 equal parts.



The point A represents the rational number $\frac{3}{5}$ and the point A′ represents the rational number $\frac{-3}{5}$.

EXERCISE 2.4

1. Express the following rational numbers in decimal form

(*a*) $\frac{47}{40}$ (*b*) $\frac{24}{25}$ (*c*) $\frac{15}{27}$ (*d*) $\frac{12}{13}$

2. Explain why 0.555 is a rational number and π is not.

3. Write any two terminating and two non-terminating rational numbers.

4. Represent each of the following rational numbers on the number line:

(*a*) $\frac{3}{5}$ (*b*) $-1\frac{1}{3}$

5. Represent $\frac{1}{5}, \frac{2}{3}, \frac{-1}{3}, \frac{-1}{5}$ on the same number line.

2.8 PROPERTIES OF REAL NUMBERS

Since, the real number system contains both the systems of rational as well as that of irrational, it is quite natural for it to exhibit their properties also. It, therefore, turns out that:

1. **Algebraic Property:** *Real numbers like rational and irrational— satisfy the commutative, associative and distributive laws for addition (+) and multiplication (×).*
2. **Closure Property:** *Real numbers are closed with respect to, addition, subtraction, multiplication and division (except by zero).* That is, if we add, subtract, multiply or divide (except by zero) two real numbers, we again get a real number.
3. **Denseness Property:** *Between any two different real numbers, there always lies a real number and hence there exist infinitely many real numbers.*
4. **Completeness Property:** *Every real number is represented by a unique point on the number line and conversely, every point on the number line represents a unique real number.*

Operations on Real Numbers

1. *The sum (difference) of a rational number and an irrational number is irrational.*
2. *The product (quotient) of a non-zero rational number with an irrational number is irrational.*

These operations are described as under:

(*a*) The sum of 2 and $\sqrt{3}$ is written as $2+\sqrt{3}$

(*b*) The product of 2 and $\sqrt{3}$ is written as $2\sqrt{3}$

(*c*) The product of $\sqrt{2}$ with $\sqrt{3}$ is written as $\sqrt{2}\cdot\sqrt{3} = \sqrt{6}$

(*d*) The quotient of 7 and $\sqrt{5}$ is written as $\frac{7}{\sqrt{5}}$.

Example 12. *Find a rational number between $\frac{2}{3}$ and $\frac{1}{6}$.*

Solution. For this we find the mean of the given rational numbers.

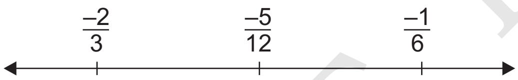
$$\left(\frac{2}{3}+\frac{1}{6}\right)\div 2 = \frac{5}{6}\div 2 = \frac{5}{6}\times\frac{1}{2} = \frac{5}{12}$$

$\frac{5}{12}$ lies between $\frac{2}{3}$ and $\frac{1}{6}$.

Hence, for any two rational numbers a and b, $\frac{a+b}{2}$ is a rational number between them.

Example 13. *Find a rational number between* $\frac{-1}{6}$ *and* $\frac{-2}{3}$.

Solution. A rational number between $\frac{-2}{3}$ and $\frac{-1}{6}$ is



$$\frac{1}{2}\left(\frac{-2}{3}+\frac{-1}{6}\right)=\frac{1}{2}\left(\frac{-4-1}{6}\right)=\frac{1}{2}\times\frac{-5}{6}=\frac{-5}{12}$$

Example 14. *Classify the following numbers as rational or irrational with justification:*

(*i*) $2-\sqrt{5}$ (*ii*) $(3+\sqrt{23})-\sqrt{23}$ (*iii*) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(*iv*) $\frac{1}{\sqrt{2}}$ (*v*) 2π.

Solution. (*i*) $(2-\sqrt{5})$, 2 is rational, $\sqrt{5}$ is irrational.

Since, the difference of a rational and an irrational is irrational.

$\Rightarrow$ $2-\sqrt{5}$ is irrational.

(*ii*) $(3+\sqrt{23})-\sqrt{23}$

Now, $3+\sqrt{23}-\sqrt{23}=3+0=3$, which is rational.

$\Rightarrow$ $(3+\sqrt{23})-\sqrt{23}$ is rational.

(*iii*) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$, which is rational.

(*iv*) $\frac{1}{\sqrt{2}}$ is the quotient of a rational and an irrational number.

Since, the quotient of a rational and an irrational number is an irrational number.

$\Rightarrow \quad \frac{1}{\sqrt{2}}$ is an irrational number.

(*v*) 2π = (rational) × (irrational)

2 is a rational number and π is an irrational number.

Since the product of a rational and an irrational is an irrational number.

$\Rightarrow \quad 2\pi$ is an irrational number.

EXERCISE 2.5

1. Insert a rational between each of given pairs of rational numbers.

(*a*) $\frac{1}{4}, \frac{2}{3}$ (*b*) $\frac{-4}{5}, \frac{1}{10}$ (*c*) $\frac{-5}{6}, \frac{-2}{5}$

2. Find three rational numbers between:

(*a*) $\frac{1}{5}$ and $\frac{4}{5}$ (*b*) $\frac{-1}{2}$ and $\frac{3}{4}$

3. Write 9 rational numbers between –1 and 2.

4. Write 10 rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$.

5. Insert 30 rational numbers between $\frac{2}{5}$ and $\frac{3}{4}$.

6. Classify the following numbers as rational or irrational with justifications:

(*a*) $\sqrt{196}$ (*b*) $3\sqrt{18}$ (*c*) $\sqrt{\frac{9}{27}}$

(*d*) $\left(1+\sqrt{5}\right)-\left(4+\sqrt{5}\right)$ (*e*) 10.124124 .... (*f*) 1.010010001 ....

2.9 APPROXIMATION

Let us consider the numbers 0.684, 9.786 and 0.00849. Now to approximate each of the numbers to two decimal places, we shall have 0.68, 9.79 and 0.01 respectively.

Example 15. *Approximate the number 87354 to the nearest*

(*i*) *10* (*ii*) *100* (*iii*) *1000*

Solution. (*i*) Since the last or ones digit is less than 5, so the second digit from the right side remains the same (*otherwise add 1 to it*) and the ones digit taken as '0'.

∴ The required number round off 10 is 87350.

(*ii*) Since the second digit is equal to 5, so the third digit from the right side will be 1 more *i.e.,* 3 + 1 = 4.

∴ The required number round off 100 is 87400.

(*iii*) Since the third digit is less than 5, so the fourth digit from the right side remains the same.

∴ The required number round off 1000 is 87000.

Rounding Off Numbers Nearest 10, 100, 1000 etc.

- Numbers equal to *or* greater than 5 are rounded up as 10.
- Numbers equal to *or* greater than 50 are rounded up as 100.
- Numbers equal to *or* greater than 500 are rounded up as 1000.
- Numbers equal to *or* greater than half of a given whole number are rounded up to that whole number, otherwise they are taken as zero.

Example 16. *Round off the number 5.3261 to the nearest*

(i) 10 *(ii) 100* *(iii) 1000*

Solution. (*i*) 5.3 (*ii*) 5.33 (*iii*) 5.326

2.10 STANDARD FORM

Consider the mass of the earth. If we write it in numerical form, we have to place 24 zeros to the right of 6 as 60,00,00,00,00,00,00,00,00,00,00,000. This makes reading the number difficult. So we express it as 6×10^{24} kg.

Expressing a number as a product of a numerical value from 1 to less than 10 and the power of 10 is called *standard form.*

For example: The speed of light can be expressed as 3×10^{8} m/s.

We can write the powers of 10 as:

$10^0 = 1$; $10^1 = 10$; $10^2 = 100$; $10^3 = 1,000$; $10^4 = 10,000$;

$10^5 = 100,000$ and so on.

In general, the standard form of a number is $x \times 10^n$.

Here, x takes values from 1 to less than 10, and n is a whole number.

We raise the power of 10 to as many times as we shift the decimal point to the left.

For example: Consider a number 380000000000 we write it as 3.8×10^{11}.

We reduce the power of 10 as many times as we shift the decimal point to the right.

For example:

We express a number 0.0052×10^6 as $5.2 \times 10^{6-3}$ or 5.2×10^3.

Example 17. *Express 63,00,000 in standard form.*

Solution. $63{,}00{,}000 = 63 \times 1{,}00{,}000 = 6.3 \times 10 \times 10^5$

$= 6.3 \times 10^6$

Example 18. *Express 3.627×10^7 in usual form.*

Solution. $3.627 \times 10^7 = \dfrac{3{,}627}{1000} \times 10^7 = \dfrac{3{,}627 \times 10^7}{10^3} = 3{,}627 \times 10^{(7-3)}$

$= 3{,}627 \times 10^4 = 3{,}627 \times 10{,}000 = 3{,}62{,}70{,}000$

Example 19. *Express 0.000061 in standard form.*

Solution. $0.000061 = \dfrac{61}{10{,}00{,}000} = \dfrac{6.1 \times 10}{10^6} = \dfrac{6.1}{10^5} = 6.1 \times 10^{-5}$

Example 20. *Express 4.23×10^{-6} in usual form.*

Solution. $4.23 \times 10^{-6} = \dfrac{423}{100} \times 10^{-6} = \dfrac{423}{10^2 \times 10^6} = \dfrac{423}{10^8} = 0.00000423$

Example 21. *Charge of an electron is 0.000000000000000000,16 coulomb. Express the charge of an electron in standard form.*

Solution. $0.000000000000000000{,}16 = 1.6 \times 10^{-19}$

Therefore, the charge of an electron is 1.6×10^{-19} coulomb.

EXERCISE 2.6

1. Approximate the following numbers to the nearest (*i*) 10 (*ii*) 100 (*iii*) 1000

(*a*) 45683 (*b*) 74004 (*c*) 0.0132 (*d*) 0.678154

(*e*) 732.548 (*f*) 0.5832

2. Express the following numbers in standard form:

(*a*) 3,65,000 (*b*) 29,00,000

(*c*) 3,70,000 (*d*) 5,02,00,00,00,00,00,000

3. Express the following numbers in usual form:

(*a*) 5.21×10^4 (*b*) 3.6983×10^7 (*c*) 6.5×10^6

4. Express the following numbers in standard form:

(*a*) 0.00000038 (*b*) 0.0045

(*c*) 0.0008 (*d*) 0.00000000000165

5. Express the following numbers in usual form:

(*a*) 3×10^4 (*b*) 3.467×10^{-3} (*c*) 8.26×10^{-6}

6. Express the number appearing in the following statements in standard form.

(*a*) The distance between the earth and the sun is 1,49,60,00,00,000 m.

(*b*) Thickness of a paper sheet is 0.0005 mm.

2.11 BINARY OPERATIONS

The word '*Bi*' means two. A binary operation on real numbers is a rule which combines two real numbers to produce a *unique real number.* A binary operation is denoted by $*$. The unique real number which a binary operation $*$ associates with two real numbers a and b is denoted by $a * b$.

If $a * b = a + b - ab$, then $*$ associates the unique real number $a + b - ab$ with a and b.

Example 22. *If* $a * b = a + b + 5$, *find* $8 * (-4)$.

Solution. Replacing a by 8 and b by (–4) in the given binary relation, we have

$$8 * (-4) = 8 + (-4) + 5 = 4 + 5 = 9.$$

Example 23. *If $m * n = 2m + n - mn$, find $5 * 3$.*

Solution. Replacing m by 5 and n by 3 in the given binary relation, we have

$$5 * 3 = 2 \times 5 + 3 - 5 \times 3$$
$$= 10 + 3 - 15$$
$$= 13 - 15 = -2.$$

EXERCISE 2.7

1. Let $*$ be a binary operation on R. Find

(*a*) $2 * 4$ if $a * b = 3a + 2b - 1$ (*b*) $3 * 2$ if $a * b = a + 3b^2$

2. Let $*$ be a binary operation on real numbers defined as $a * b = (2a - b)^2$. Find $3 * 5$ and $5 * 3$. Is $3 * 5 = 5 * 3$?

3. If $a * b = 2a + 5b$, find $4 * 3$ and $3 * 4$. Is $4 * 3 = 3 * 4$? Is the operation $*$ commutative?

4. If $m * n = m^2 - mn + n^2$, find $2 * 6$ and $6 * 2$. Is $2 * 6 = 6 * 2$? Is the operation $*$ commutative.

REVIEW EXERCISE

1. List five rational numbers.

2. What will be the additive inverse of the rational number $\frac{5}{7}$?

3. Find:

(*a*) $\frac{-3}{4} \times \frac{1}{7}$ (*b*) $\frac{2}{3} \times \frac{-5}{9}$.

4. Write:

The rational number that is equal to its negative.

5. Name the property under multiplication used in $\frac{-19}{29} \times \frac{29}{-19} = 1$.

6. Multiply $\frac{6}{13}$ by the reciprocal of $-\frac{7}{16}$.

7. By what rational number should $\frac{-33}{8}$ be multiplied to get $\frac{-11}{2}$?

8. Express the following rational numbers in decimal forms:

(*a*) $\frac{42}{9}$ (*b*) $\frac{-1}{37}$.

9. Represent the following rational number on the number line $2\frac{3}{4}$.

10. Classify the following numbers as rational or irrational with justifications:

(*a*) $\frac{\sqrt{28}}{\sqrt{343}}$ (*b*) $-\sqrt{0.4}$ (*c*) $\frac{\sqrt{12}}{\sqrt{75}}$ (*d*) 0.5918

11. Approximate the following numbers to the nearest (*i*) 10 (*ii*) 100 (*iii*) 1000

(*a*) 618712 (*b*) 23871 (*c*) 584.732 (*d*) 19.8972

12. Express the number appearing in the following statements in standard form.

(*a*) The speed of light is 30,00,00,000 m/s.

(*b*) Radius of a red blood cell is 0.000003 mm.

13. Let $*$ be a binary operation on R. Find $6 * 4$ if $a * b = \text{LCM}(a, b)$.

MULTIPLE CHOICE QUESTIONS (MCQs)

1. A rational number can be represented in the form of:

(*a*) $\frac{p}{q}$ (*b*) pq (*c*) $p + q$ (*d*) $p - q$

2. The value of $\frac{1}{2} \times \frac{3}{5}$ is equal to:

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{2}{5}$

3. The value of $\frac{1}{2} \div \frac{3}{5}$ is equal to:

(*a*) $\frac{3}{10}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{6}{5}$ (*d*) $\frac{5}{6}$

4. The value of $\frac{1}{2} + \frac{1}{4}$ is equal to:

(*a*) $\frac{3}{4}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) 1

5. The value of $\frac{5}{4} - \frac{8}{3}$ is:

(*a*) $\frac{17}{12}$ (*b*) $-\frac{17}{12}$ (*c*) $\frac{12}{17}$ (*d*) $-\frac{12}{17}$

6. The associative property is applicable to:

(*a*) Addition and subtraction (*b*) Multiplication and division

(*c*) Addition and multiplication (*d*) Subtraction and division

7. The multiplicative identity of a rational number is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) –1

8. What is the product of $\frac{2}{9}$ and $\frac{3}{4}$?

(*a*) 1/6 (*b*) 2/3 (*c*) 1/9 (*d*) 1/4

9. What is the reciprocal of 1/9?

(*a*) 9 (*b*) 0 (*c*) 1 (*d*) None of these

10. What is the value of 100 divided by 0?

(*a*) 0 (*b*) 100 (*c*) 1 (*d*) Undefined

11. How many rational numbers are there between $\frac{3}{4}$ and 1?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) Countless

12. The operation ∗ defined by $a * b = a + b - ab$, calculate 3 ∗ 5

(*a*) –1 (*b*) –2 (*c*) –4 (*d*) –7

13. Which rational number has no multiplicative inverse?

(*a*) 1 (*b*) 0 (*c*) –1 (*d*) None of these

14. Additive inverse of $\frac{-2}{3}$ is:

(*a*) $\frac{-3}{2}$ (*b*) $\frac{2}{3}$ (*c*) 0 (*d*) $\frac{3}{2}$

15. A rational number between $\frac{1}{2}$ and $\frac{-2}{3}$ is:

(*a*) $\frac{-1}{12}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{7}{6}$ (*d*) 1

16. Multiplicative inverse of $\frac{-5}{-7}$ is:

(*a*) $\frac{-5}{7}$ (*b*) $\frac{5}{7}$ (*c*) $\frac{7}{5}$ (*d*) $\frac{-7}{5}$

RECAP AT A GLANCE

- A rational number is defined as *a number that can be expressed in the form* $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
- The product of two negative numbers is a positive number.
- The product of a negative and a positive number is a negative number.
- The product of two positive numbers is a positive number.

- Rational numbers are *closed* under multiplication.
- Multiplication is *associative* for rational numbers.
- Multiplication is *commutative* for rational numbers.
- 1 is the multiplicative *identity* for rational numbers.
- The product of a rational number and its multiplicative inverse is 1.
- 0 has no multiplicative inverse.
- The product of a rational number with its reciprocal is always 1.
- Every real number is either a rational number or an irrational number.
- Binary operation on real numbers is a rule which combines two real numbers to produce a *unique real number.*

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